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THE IMPACT ON FLOATS OR HULLS DURING LANDING AS AFFECTED BY BOTTOM WIDTH

By E. Mewes

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THE IMPACT ON FLOATS OR HULLS DURING LANDING AS AFFECTED BY BOTTOM WIDTH*

By E. Mewes

According to the theoretical computations given here, there is an increase in the impact during the landing of seaplanes with increase in bottom width only up to a certain limiting value of the bottom width. This limiting value both for straight V and curved V bottoms is independent of the magnitude of the keel angle and is given by the following simple expression:

$$\frac{G_{red}}{B_g{}^2\ L_{max}} = 1{,}960\ \frac{kg}{m^3} \quad \text{or} \quad B_g = \sqrt{\frac{G_{red}}{1.96\ \gamma_w\ L_{max}}}$$

where

G_{red} is the reduced weight at impact position

B_g, computed limiting value for the bottom width

L_{max}, maximum impact length

In most cases occurring in practice this value is usually exceeded.

OBJECT OF THIS PAPER

In the design of flying boats and seaplane floats, an important question that arises is the proper choice of the best width for the hull or float bottoms. This choice is influenced by several factors and, besides considerations of the hydrodynamical take-off performance, there is also to be taken into account the necessary weight to insure the required strength of structure. The effect of the bottom width during the landing impact may be quickly computed under somewhat idealized assumptions.

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*"Über den Einfluss der Bodenbreite eines Schwimmers oder Flugbootes auf den Landestoss." Luftfahrtforschung, vol. 13, no. 5, May 20, 1936, pp. 148-154.

If the bottom width approaches zero $B \longrightarrow 0$, the impact force likewise approaches zero $P \longrightarrow 0$. For finite widths the impact forces are finite. As the width increases, the impact must at first increase steadily. The theoretical computations that are here given, show that this increase does not go on indefinitely but that a value for the bottom width is reached to which there corresponds the maximum impact force for the same landing conditions. This width B_g, herein often denoted briefly as the limiting width, is the one that we seek to determine as a function of the other float variables. In addition, there will also be indicated the effect of varying the width above and below this limiting value on the maximum value of the impact force.

COMPUTATION

The problem investigated is the force on a V-shape bottom during impact on water. (See fig. 1.) The underlying principle for the computation is the theorem of conservation of momentum:

$$\int P \, dt = M \, v - M_o \, v_o$$

This theorem is applied both to the float or hull - the force on which is denoted by P' - and to the fluid, on which the resulting force is P''. By the principle of action and reaction, we therefore have:

$$P' = - P''$$

$$\left| P' \right| = \left| P'' \right| = P$$

For the hull or float we have:

$$\int P' \, dt = M_r \, (v_n - v_o)$$

where v_o is the downward velocity at the instant of first contact.

For the fluid we have:

$$P'' \, dt = M_w \, v_n - 0 \, v_o$$

where M_w is the so-called "accelerated mass of water."

In the theoretical impact, computations of von Kármán (reference 1) and Wagner (reference 2), in which the effect of the finite length of bottom on the impact was neglected, the magnitude of the accelerated water mass is the mass of water whose volume is that of a half-cylinder having a diameter equal to the impact width.

$$M_w = \frac{1}{2} \pi \rho_w c^2 L = f(c)$$

where c denotes half the wetted width (fig. 1). For $c = 0$, $M_{w_0} = 0$, and therefore the second term on the right-hand side of the momentum equation for the fluid vanishes. We further introduce the ratio:

$$\mu = \frac{M_w}{M_r}$$

$$\mu = \frac{\rho_w \pi L}{2M_r} c^2 = \frac{\pi}{2} \frac{\gamma_w L}{\tau G} c^2$$

where $\tau = \frac{M_r}{M} = \frac{G_{red}}{G}$ is a mass reduction factor.

We thus obtain the equations:

$$M_r (v_n - v_o) = - M_w v_n$$

$$(M_r + M_w) v_n = M_r v_o$$

$$v_n = \frac{M_r}{M_r + M_w} v_o$$

$$\frac{v_n}{v_o} = \frac{1}{1 + \mu}$$

Now
$$v_n = \frac{dy}{dt}$$

and the ratio of $\frac{dy}{dt}$ to $\frac{dc}{dt}$ Wagner denoted by u:

$$u = \frac{\frac{dy}{dt}}{\frac{dc}{dt}}$$

so that
$$\frac{dc}{dt} = \frac{1}{u} v_n$$

The impact force on the bottom is:

$$P = M_r \; b$$

where the acceleration is b.

$$b = -\frac{dv_n}{dt}$$

$$= -\frac{dv_n}{d\mu}\frac{d\mu}{dc}\frac{dc}{dt} = -\frac{dv_n}{d\mu}\frac{d\mu}{dc}\frac{1}{u}v_n$$

By substituting the values of $v_n = f(\mu)$ and $\mu = f(c)$, there is obtained:

$$b = \frac{\pi \; \rho_w \; L}{M_r} v_o^2 \; \frac{c}{u}\left(\frac{1}{1+\mu}\right)^3$$

$$P = \pi \; \rho_w \; L \; v_o^2 \; \frac{c}{u}\left(\frac{1}{1+\mu}\right)^3$$

with $$\mu = \frac{\pi \; \rho_w}{2M_r} L \; c^2$$

Thus P may be computed for every value of L and c as soon as u is known as a function of c. This function $u = f(c)$ is determined by the bottom shape.

COMPUTATION FOR STRAIGHT V-BOTTOM

For the straight bottom, which we shall investigate first,

$$y = \beta \; x$$

From figure 1 it is seen that $c > x$.

Assuming that the two-dimensional flow pattern about a flat plate is also applicable to this V-shape bottom, then according to Wagner, for the straight bottom

$$u = \frac{2}{\pi}\beta = \text{const.}$$

and this value for u is substituted in the equation just derived for the impact force:

$$P = \frac{1}{2}\,\pi^2\,\rho_w\,L\,v_o^2\,\frac{1}{\beta}\,c\,\left(\frac{1}{1+\mu}\right)^3$$

with

$$\mu = \frac{\pi}{2}\,\frac{\rho_w}{M_r}\,L\,c^2$$

and thus the impact force is given for every value of c and L.

We shall first extend our computations to the case of a float which lands vertically on the water, the length L of the bottom remaining constant. We seek to determine the value of c which gives a maximum value for the impact force P_{max}. The maximum value of the impact force occurs either when

a) $c = B/2$ or

b) at the instant when in the above equation for the impact force $\frac{\partial P}{\partial c} = 0$.

As long as $\frac{\partial P}{\partial L} = 0$ has no solution within the range

$$0 \leqq c \leqq \frac{B}{2}$$

the maximum value occurs at the instant of complete wetting of the bottom ($c = B/2$). In that case an increase in the impact force is to be expected with an increase in the bottom width. In case b), however, the maximum value of the impact force is reached even before the bottom is entirely immersed so that after a certain value is reached increasing the bottom width is no longer followed by an increase in the impact force. In all cases included under b) the maximum value of the impact force, for a constant length of bottom, is independent of the width.

We shall now consider those cases under b), setting the derivative $\partial P/\partial c$ equal to zero:

$$\frac{\partial P}{\partial c} = 2\left(\frac{\pi}{2}\right)^2 \rho_w\; v_o^2\;\frac{1}{\beta}\;L\;\frac{\partial}{\partial c}\left[c\left(\frac{1}{1+\mu}\right)^3\right] \text{ with } \mu = \frac{\pi}{2}\,\frac{\rho_w}{M_r}\,L\,c^2$$

$$\frac{\partial P}{\partial c} = 0 \qquad \text{for} \qquad \frac{\partial}{\partial c}\left[c\left(\frac{1}{1+\mu}\right)^3\right] = 0$$

i.e., for $\mu = \frac{1}{5}$.

For a straight-keeled bottom of given length and having a sufficient width - that is, $B/2$ being greater than the value of c computed from the equation $\partial P/\partial c = 0$, the greatest impact force occurs for a value of $\mu = 1/5$; or for

$$c = \sqrt{\frac{2}{\pi} \frac{M_r}{\rho_w L} \frac{1}{5}}, \qquad c = 0.357 \sqrt{\frac{G_{red}}{\gamma_w L}}$$

For a symmetrical landing G_{red} is approximately equal to G, for central float seaplanes and flying boats, and approximately equal to $G/2$ for twin-float seaplanes and twin flying boats. For this particular value of c, we substitute $B_g/2$ where B_g represents the limiting value of B above which, for a given length of float or hull, there is no increase in the impact force with increasing width:

$$\frac{B_g}{2} = \sqrt{\frac{1}{5} \frac{2}{\pi} \frac{G_{red}}{\gamma_w L}}$$

$$B_g = 0.713 \sqrt{\frac{\tau G}{\gamma_w L}}$$

$$B_g \approx 0.713 \sqrt{\frac{G}{L}}$$

(G in t, B_g and L in m)

For the single-float seaplane or flying boat:

$$B_g = 0.713 \sqrt{\frac{G}{\gamma_w L}}$$

and for a twin-float seaplane or twin flying boat:

$$B_g = 0.504 \sqrt{\frac{G}{\gamma_w L}}$$

$$\frac{G_{red}}{\gamma_w B_g^2 L} = 1.96, \qquad \frac{G_{red}}{\gamma_w B_g^3} = 1.96 \frac{L}{B_g}$$

It may be seen from these equations that different types of seaplanes and flying boats show similar relations with respect to the impact force if they have equal values for $G/B^2 L$, whereas the characteristic G/B^3 commonly used in float design depends on L/B.

The total impact force on a V-shape float or hull bottom with given impact length L is increased with increase in bottom width as long as $B < B_g$ and is equally large for different bottom widths if

$$B \geqq B_g \quad \text{(See fig. 4.)}$$

The limiting value of B is, according to the formulas derived, independent of the angle of the V. (P_{max} itself does, however, depend on this angle.)

It was assumed that the impact length of the float was constant. During the landing of seaplanes different values of L up to a maximum are possible:

$$0 \leqq L \leqq L_{max}$$

When there is a sharp curvature of the bottom surface or the surface of the water (short waves), first contact occurs at a point $(L \longrightarrow 0)$. During the downward motion the wetted length may be increased somewhat although it may still remain relatively quite small up to the instant when the maximum impact force is attained. Beyond a certain limit (length of float body) the impact length cannot increase. From this consideration it may be seen that the wetted length lies below a certain upper limit but just where this limit lies may be estimated only approximately at present. Up to the present the maximum impact length was determined by shaping the float to fit the wave form. For a smooth water surface this maximum impact length L_{max} is obtained by drawing the tangent at the bottom in front of the step and estimating the length so as to have an approximate agreement of the keel line with this tangent. During the downward motion in the water the impact length may become somewhat greater. In the theoretical computations it is assumed that the length remains constant during the immersion.

It is not obvious at the outset whether the maximum wetted length corresponds also to the maximum impact force. The following two cases are to be distinguished: The maximum impact force occurs either at

a) the maximum length L_{max}, or for

b) $\frac{\partial P}{\partial L} = 0$, where the length corresponding to the maximum impact force lies within the range

$$0 \leqq L \leqq L_{max}$$

Case a) enters into consideration only when the equation $\frac{\partial P}{\partial L} = 0$ has no solution for L within the range $0 < L < L_{max}$. In that case the maximum value of the impact force will be obtained for the maximum possible impact length L_{max} and the conditions previously derived for the maximum impact force at various wetted widths are in general valid for all forms of floats if for L we substitute L_{max}.

There is still to be investigated, however, the case where the maximum impact occurs at smaller values of the impact lengths. To obtain these $\partial P/\partial L$ is set equal to zero. We have:

$$\frac{\partial P}{\partial L} = 2\left(\frac{\pi}{2}\right)^2 \rho_w \; v_o^2 \; \frac{1}{\beta} \; c \; \frac{\partial}{\partial L}\left[L\left(\frac{1}{1+\mu}\right)^3\right]$$

with $$\mu = \frac{\pi}{2}\frac{\rho_w}{M_r} c^2 L$$

$$\frac{\partial P}{\partial L} = 0$$

for $$\frac{\partial}{\partial L}\left[L\left(\frac{1}{1+\mu}\right)^3\right] = 0$$

that is, for $\mu = \frac{1}{2}$.

The corresponding length is:

$$L = \frac{2}{\pi}\frac{M_r}{\rho_w}\frac{\mu}{c^2}$$

$$L = \frac{M_r}{\pi \; \rho_w \; c^2} = \frac{\tau \; G}{\pi \; \gamma_w \; c^2}$$

There thus corresponds to each value of c, a length L at which the maximum impact force occurs. We must find the range of widths within which such a value of $L < L_{max}$ may occur. We thus have:

$$\frac{M_r}{\pi\ \rho_w\ c^2} < L_{max}$$

$$c > \sqrt{\frac{M_r}{\pi\ \rho_w\ L_{max}}}$$

Since $$c \leqq \frac{B}{2}$$

always, to satisfy the above condition, we must have:

$$\frac{B}{2} > \sqrt{\frac{M_r}{\pi\ \rho_w\ L_{max}}}$$

Substituting the limiting value, denoted by B_l, we obtain:

$$\frac{B_l}{2} = \sqrt{\frac{1}{\pi}\ \frac{\tau\ G}{\gamma_w\ L_{max}}}$$

$$B_l = 1.126 \sqrt{\frac{\tau\ G}{\gamma_w\ L_{max}}}$$

$$B_l = 1.126 \sqrt{\frac{G_{red}}{L_{max}}}$$

(G_{red} in t, B and L_{max} in m)

$$\frac{G_{red}}{\gamma_w\ B_l^2\ L_{max}} = 0.79$$

The possibility, therefore, that the maximum impact force does not correspond to the maximum impact length occurs only at the greater bottom widths: $B_l > B_g$.

We shall now see how the maximum impact force changes when $B > B_l$ and $L < L_{max}$. Substituting

$$L = \frac{M_r}{\pi\ \rho_w\ c^2}$$

and $$\mu = \frac{1}{2}$$

in the equation for the impact force:

$$P = \frac{1}{2}\,\pi^2\,\rho_w\,v_o^2\,\frac{1}{\beta}\,c\,L\,\left(\frac{1}{1+\mu}\right)^3$$

we obtain for the maximum impact force for any value of the width within the range $B \geqq B_l$ the expression

$$P_{(max)} = \left(\frac{2}{3}\right)^3\,\frac{\pi}{2}\,M_r\,v_o^2\,\frac{1}{\beta}\,\frac{1}{c}$$

Smaller maximum values for the impact force correspond to greater wetted widths (2c). The maximum for all impact forces P_{max}, however, is reached for the value $2c = B_g$ and remains unchanged for $2c > B_l$.

The impact force as a function of c, the wetted half width, has been worked out in a numerical example and the results are shown in figure 3. During the entire downward motion $e = \frac{P}{G} = f(c)$ is plotted with the value of L_{max}, and moreover, for $L < L_{max}$ the curve has been plotted using the value of L obtained from the equation $\frac{\partial P}{\partial L} = 0$. It may be seen that there are no values of $L < L_{max}$ which give the highest value for the impact force. This maximum value always occurs for the maximum wetted length of float L_{max}. The maximum impact force occurs for a value of $c = B_g/2$ and is independent of the width. Figure 4 shows the maximum impact force-to-weight ratios $e_{max} = \frac{P_{max}}{G}$, plotted against the width B, for the same numerical example.

These computations for the straight V-bottom have been gone into in detail because up to the present the strength computations for float and hull bottoms have been made exclusively on equivalent straight V bottoms.

The effect of the V angle on the impact force will be considered in another report. The following formulas are based on Wagner's theoretical computations where the elasticity of the construction is not taken into account. The value for the maximum impact force for straight V-bottom floats for all widths may, according to the theory of Wagner, be given by the equation:

$$P_{max} = \left(\frac{\pi}{2}\right)^2\,\rho_w\,v_o^2\,\frac{1}{\beta}\,\left(1 - \sqrt[3]{0.1\,\beta^2}\right)\,B_h\,L_{max}\,\left(\frac{1}{1+\mu_h}\right)^3$$

with
$$\mu_h = \frac{\pi}{2} \frac{\rho_w\ L_{max}}{M_r} \left(\frac{B_h}{2}\right)^2$$

The value of B_h to be used is indicated below:

a) For very narrow bottoms within the range

$$B \leqq B_g = 2 \sqrt{\frac{1}{5} \frac{2}{\pi} \frac{M_r}{\rho_w\ L_{max}}} \quad \text{or} \quad \frac{G_{red}}{\gamma_w\ B^2\ L_{max}} \geqq 1.96$$

B_h is to be substituted for B so that we obtain:

$$P_{max} = \left(\frac{\pi}{2}\right)^2 \rho_w\ v_o^2\ \frac{1}{\beta}\ (1 - \sqrt[3]{0.1\ \beta^2})\ B\ L_{max} \left(\frac{1}{1 + \mu}\right)^3$$

with
$$\mu = \frac{\pi}{2} \frac{\rho_w\ L_{max}}{M_r} \left(\frac{B}{2}\right)^2$$

In these cases $\mu < \frac{1}{5}$. Neglecting μ in comparison with l in the term $\left(\frac{1}{1 + \mu}\right)^3$, P_{max} becomes simply proportional to B.

The apprcximation

$$P_{max} \approx \left(\frac{\pi}{2}\right)^2 \rho_w\ v_o^2\ \frac{1}{\beta}\ (1 - \sqrt[3]{0.1\ \beta^2})\ B\ L_{max}$$

agrees with the accurate expression only within the range $B < \frac{1}{3} B_g$ as shown in figure 4. The factor $\left(\frac{1}{1 + \mu}\right)^3$ which takes into account the decrease of the downward velocity from the moment of first contact up to the time of maximum impact, has quite a considerable effect within the range $\frac{1}{3} B_g \leqq B \leqq B_g$.

b) For sufficiently wide bottoms, in the range

$$B \geqq B_g = 2 \sqrt{\frac{1}{5} \frac{2}{\pi} \frac{M_r}{\rho_w\ L_{max}}} \quad \text{or} \quad \frac{G_{red}}{\gamma_w\ B^2\ L_{max}} \leqq 1.96$$

we must substitute

$$B_h = B_g$$

and
$$\mu_h = \frac{1}{5}$$

We then have:

$$P_{max} = 2 \sqrt{\frac{1}{5}\left(\frac{\pi}{2}\right)^3 \rho_w M_r L_{max} \frac{1}{\beta} (1 - \sqrt[3]{0.1 \beta^2})} v_o^2 \ 0.577$$

$$= 1.015 \frac{1}{\beta} (1 - \sqrt[3]{0.1 \beta^2}) v_o^2 \sqrt{\rho_w M_r L_{max}}$$

$$e_{max} = 1.015 \frac{1}{\beta} \left(1 - \sqrt[3]{0.1 \beta^2}\right) \frac{v_o^2}{g} \sqrt{\gamma_w \frac{L_{max}}{G} \tau}$$

The numerical coefficient should be 1.015 and not 0.835 as is given, for example, in the Zeitschrift für Flugtechnik und Motorluftschiffahrt, vol. 22, no. 1, 1931, page 7. The factor $(1 - \sqrt[3]{0.1 \beta^2})$ corresponds approximately to the Wagner correction factor P_W/P for the finite angle of a straight V bottom.

The results show that for sufficiently large bottom widths, the greatest impact force is attained as soon as the velocity becomes 0.833 times the initial impact velocity. The impact force is thus smaller by 42.3 percent than the computed value, assuming the velocity to remain constant ($\mu = 0$).

COMPUTATIONS FOR CURVED V BOTTOMS

Generally a V shape is given to the planing bottom of a float in front of the main step, the float being rather sharp at the keel and curving outward in such a manner as to obtain a good spray pattern. The computations in this section are based on a float bottom having small curvature (fig. 5). The bottom plan forms that are much in use are straight from the keel on for a large part of their width and strongly curved ahead of the chine. For these bottoms the maximum impact force on landing occurs when the entire bottom width is wetted, and this also is true for the example we shall now investigate, so that there is an essential difference between this case and the straight V-bottom example we have just investigated.

The simplicity of the treatment of straight V bottoms was due to the fact that the same conditions applied during impact for both narrow and wide bottoms, so that smaller maximum impact forces could not occur for greater widths of this type of bottom. The behavior of wide curved bottoms is, on the contrary, not so easily deducible from that of narrow curved bottoms.

In order to compare curved bottoms of different widths we shall assume, not that the same equation $y = f(x)$ for the bottom curve holds for all widths, but that there is similarity between them (see fig. 6b) so that in nondimensional representation we have for all widths

$$\eta = f(\xi)$$

where
$$\eta = \frac{y}{B/2}$$

and
$$\xi = \frac{x}{B/2}$$

In this case, too, the impact force must approach zero as $B \longrightarrow 0$. At very small finite widths the maximum impact force will always occur at the end of downward motion of the bottom $(s = 1)$. (Here, too, the nondimensional representation $\left(s = \frac{c}{B/2}\right)$ is used instead of the Wagner notation.) This was also true for the straight bottom. For the bottom shape of constant downward curvature there is even a greater tendency for the impact force to increase when there is a large immersion in the water. The factor $1/u$ that affects the impact force is then no longer constant but in general increases very greatly as $s \longrightarrow 1$. Equations

$$P = \frac{\pi}{2} \rho_w v_o^2 L \cdot B \frac{s}{u} \left(\frac{1}{1 + \mu}\right)^3$$

with
$$\mu = \frac{\pi}{2} \frac{\rho_w}{M_r} L \left(\frac{B}{2}\right)^2 s^2 = C s^2$$

and
$$u = \frac{2}{\pi} \beta_0 + \beta_1 \frac{B}{2} s + \frac{4}{\pi} \beta_2 \left(\frac{B}{2}\right)^2 s^2$$

for
$$\eta = \beta_0 \xi + \beta_1 \frac{2}{B} \xi^2 + \beta_2 \left(\frac{2}{B}\right)^2 \xi^3$$

yield the maximum value for the impact force for $s = 1$ with the corresponding $u_a = \text{const.}$, namely,

$$P_{max} = \frac{\pi}{2} \rho_w v_o^2 L_{max} B \frac{1}{u_a} \left(\frac{1}{1 + \mu_a}\right)^3$$

with
$$\mu_a = \frac{\pi}{2} \frac{\rho_w}{M_r} L_{max} \left(\frac{B}{2}\right)^2$$

The fact that L_{max} is to be substituted in the equation follows from the considerations on the previous example. Within this range P_{max} depends on L_{max} and B.

The impact force increases with the width. The maximum of all impact forces is obtained by setting $\frac{\partial P_{max}}{\partial B} = 0$. This equation gives the same limiting value for the width B_g as for the case of straight V bottoms:

$$B_g = 2 \sqrt{\frac{1}{5} \frac{2}{\pi} \frac{M_r}{\rho_w L_{max}}}$$

or

$$\frac{G_{red}}{\gamma_w B_g^2 L_{max}} = 1.96$$

Figure 7 shows the variation of the impact force worked out for an example with B_g as the width. For bottoms of greater widths the maximum values of the impact forces are smaller. For widths that are not too large the maximum value of the impact force occurs at the end of the immersion of the boat bottom. As long as the limiting value B_l is not reached, the maximum impact force occurs at the maximum possible wetted length. Figure 7 also shows the variation in the impact force for B_l and L_{max}.

Above the value B_l the maximum impact force occurs not at the maximum wetted length L_{max} but at a smaller wetted length, namely,

$$L = \frac{M_r}{\pi \rho_w c^2}$$

and the maximum impact force becomes independent of L_{max}. In figure 7 are given the maximum values of $e = \frac{P}{G}$ for $\frac{B}{2} = 1$ m and also for $\frac{B}{2} = 1.32$ m. In these cases, too, it may be seen that the maximum impact force occurs at the maximum downward motion and is therefore independent of B. With increasing width the force decreases in accordance with the relation:

$$P_{max} = \left(\frac{2}{3}\right)^3 M_r v_o^2 \frac{1}{u_a \frac{B}{2}}$$

Even for the relatively very large width of $B = 3$ m, the maximum value of the impact force for this small curvature bottom occurs at the end of the immersion. (See fig. 7.)

Only at very large values of the width does the maximum value of the impact force occur before the bottom is completely immersed. An example was computed for $\frac{B}{2} = 2.0$ m and plotted on figure 7.

Figure 8 summarizes the results and shows the variation of the ratio $e_{max} = \frac{P}{G}$ with bottom width for the type of bottom considered. The limiting values B_g and B_l are valid in general for bottoms of any form. From the theoretical computations of Wagner on the impact of seaplanes, the result is obtained that beyond a certain limiting value for the widths, there is no longer any further increase in the maximum value of the impact force with increasing bottom widths. The limiting value for the bottom width appears from these computations to be independent of the keel angle provided the conditions assumed by Wagner are fulfilled, namely, that the keel angle must not be either too small or too large $(\beta \longrightarrow 0)$ since in the latter case the elastic yielding of the construction comes into effect. The limiting value for the width may be determined from the following expressions:

$$B_g = 2\sqrt{\frac{1}{5}\,\frac{2}{\pi}\,\frac{G_{red}}{\gamma_w\ L_{max}}}$$

$$= 0.713\sqrt{\frac{G_{red}}{L_{max}}} \quad (G_{red} \text{ in t, } L_{max} \text{ and } B_g \text{ in m})$$

$$\frac{G_{red}}{\gamma_w\ B_g^2\ L_{max}} = 1.96$$

for $\tau = 1$.

For the flying boat or single-float seaplane:

$$\frac{G}{\gamma_w\ B_g^3} = 1.96\ \frac{L_{max}}{B_g}$$

and for the twin-float seaplane or twin flying boat

$$\frac{G/2}{\gamma_w\ B_g^3} = 1.96\ \frac{L_{max}}{B_g}$$

The usual values for $\frac{G}{\gamma_w B^3}$ for actual seaplanes lie between 0.4 and 2, and therefore the corresponding range within which P_{max} becomes independent of B, is

$$\frac{L_{max}}{B} > \frac{1}{5} \text{ to } 1$$

This condition is almost always satisfied so that Wagner's expressions for straight-bottom floats or hulls are:

$$P_{max} = \sim v_o^2 \quad \frac{1}{\beta}\left(1 - \sqrt[3]{0.1 \ \beta^2}\right)\sqrt{\rho_w \ L_{max} \ M_r}$$

$$= 3.28 \ v_o^2 \ \frac{1}{\beta}\left(1 - \sqrt[3]{0.1 \ \beta^2}\right)\sqrt{G_{red} \ L_{max}}$$

$$e_{max} = 3.28 \ v_o^2 \ \frac{1}{\beta}\left(1 - \sqrt[3]{0.1 \ \beta^2}\right)\sqrt{\tau \ \frac{L_{max}}{G}}$$

(G_{red} and P in kg; L_{max} in m; v_o in m/s)

For curved bottoms in the majority of cases (for

$$B \doteq B_l = 2 \sqrt{\frac{G_{red}}{\pi \ \gamma_w \ L_{max}}} \quad \text{or} \quad \frac{G_{red}}{\gamma_w \ B^2 \ L_{max}} \leqq 0.79;$$

that is, for not very large widths at which the maximum impact force would occur before the bottom is completely wetted) the equations are:

$$P_{max} = \left(\frac{2}{3}\right)^3 M_r \ v_o^2 \ \frac{1}{u_a \ \frac{B}{2}}\left(1 - \frac{\beta_a}{\pi} - \sqrt{0.06 \ u_a}\right)$$

$$e_{max} = 0.06 \ \tau \ v_o^2 \ \frac{1}{u_a \ B}\left(1 - \frac{\beta_a}{\pi} - \sqrt{0.06 \ u_a}\right)$$

with

$$u_a = \frac{2}{\pi} \beta_o + \beta_1 \frac{B}{2} + \frac{4}{\pi} \beta_2 \left(\frac{B}{2}\right)^2 + \ldots \quad \text{(according to Wagner)}$$

when the equation for the float bottom is assumed to be of the form

$$y = \beta_o \ x + \beta_1 \ x^2 + \beta_2 \ x^3 + \ldots$$

or with

$$u_a = \frac{2}{\pi} \beta_i - k_n \beta_n \quad \text{(according to Weinig)}$$

and $$k_n \approx 0.793 \sqrt{n - 0.4}$$

when the equation of the float bottom is of the form

$$\eta = \beta_i \xi - \beta_n \xi^n$$

The factor $\left(1 - \frac{\beta_a}{\pi} - \sqrt{0.06 u_a}\right)$ corresponds approximately to the correction factor of Wagner:

$$\frac{P_W}{P} = 1 - \frac{\beta}{\pi} - 0.15 \frac{u}{\pi} - \frac{u}{\pi} \ln \frac{1}{u}$$

for the outer edge and β_a corresponds to β for $s = 1$. When the bottom is nearly horizontal at the chine, then β_a is approximately zero in the above formula. Besides being dependent on u_a which is largely conditioned by the bottom shape, the impact force depends considerably on the width, decreasing with increasing width. This is true only of symmetrical landing, assuming that the other variables determining the magnitude of the impact are independent of the width. The behavior of the seaplane after impact, which behavior is often of equal importance for determining the stresses at take-off and landing, is not touched upon here.

CONCLUSION

For floats and hulls having V bottoms the impact force does not necessarily increase with increasing width. Therefore, the weight of the float landing gear, side walls, and other parts, and of the fuselage construction need not be increased with increasing bottom width, but the weight of the bottom construction itself, on the other hand, does increase with increase in bottom width and is determined largely by the type of construction. These relations have not yet been closely investigated.

Translation by S. Reiss,
National Advisory Committee
for Aeronautics.

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1. von Kármán, Th.: The Impact on Seaplane Floats During Landing. T.N. No. 321, N.A.C.A., 1929.

2. Wagner, H.: Über Stoss- und Gleitvorgänge an der Oberfläche von Flüssigkeiten. Z.f.a.M.M., vol. 12, 1932, p. 193.

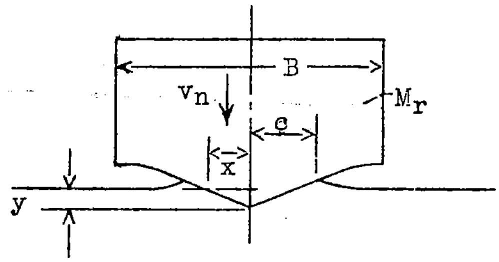


Figure 1.- V-shape bottom landing on water.

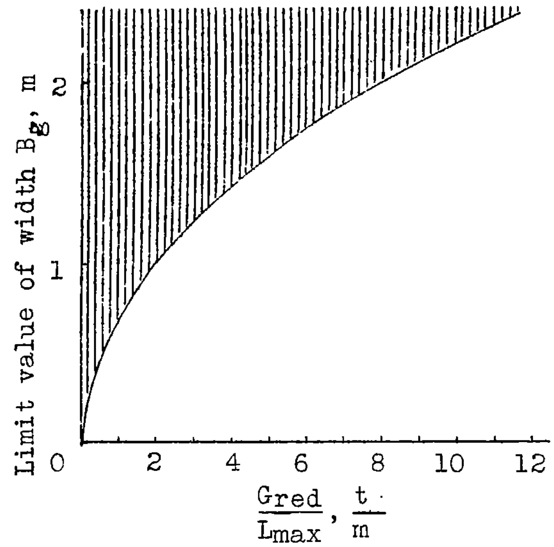


Figure 2.- Limiting value of bottom width B_g beyond which there is no increase in impact force.

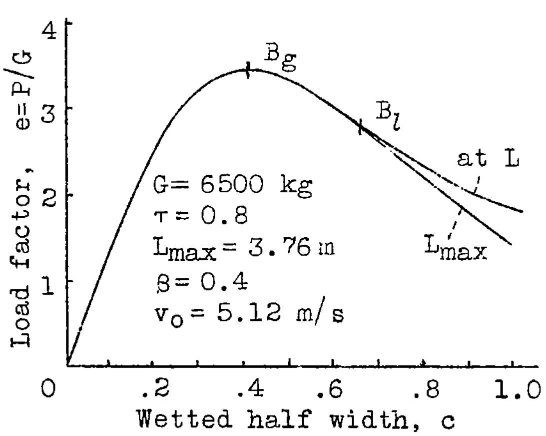


Figure 3.- Variation of impact force with wetted half-width for straight V bottom of sufficient width.

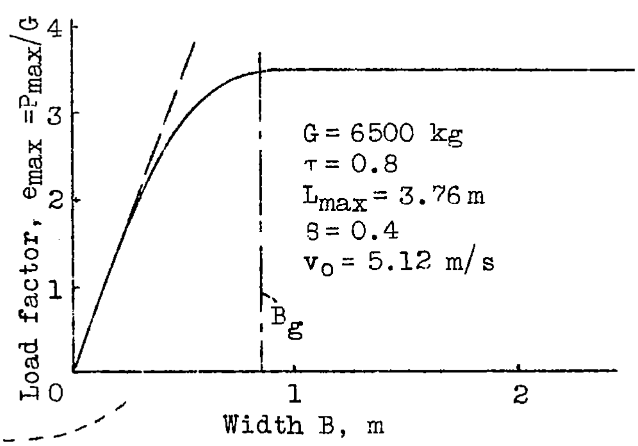


Figure 4.- Impact force as function of float width for straight V bottom.

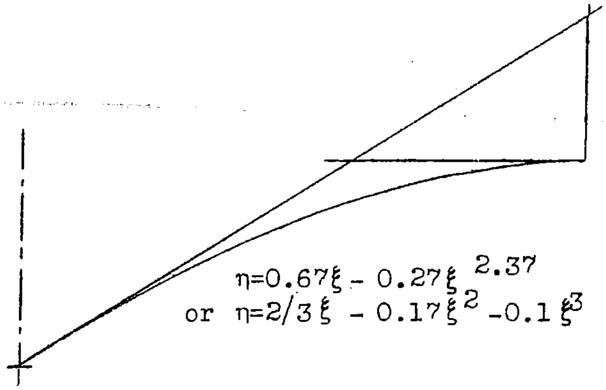
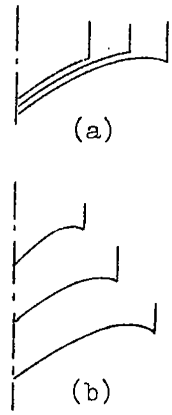


Figure 5.- Half section of float considered in the example



a. Bottom curves having same equation $y=f(x)$
b. Bottoms of various widths but having similar shapes

Figure 6.- Curved bottoms of various widths.

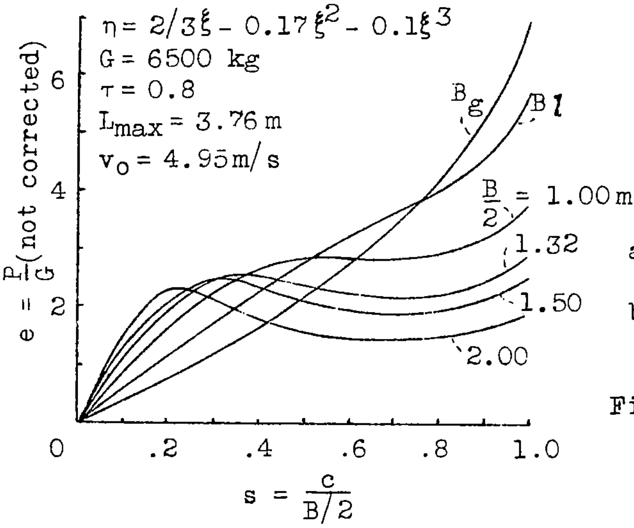


Figure 7.- Impact forces on curved bottoms.

$\eta = 2/3\xi - 0.17\xi^2 - 0.1\xi^3$
G = 6500 kg
τ = 0.8
L_{max} = 3.76 m
v_o = 4.95 m/s
Impact loads not corrected.

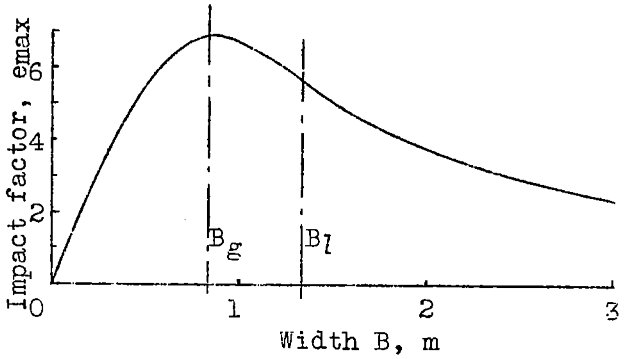


Figure 8.- Impact force as function of bottom width for curved bottoms.